RANDOM MATRIX THEORY AND ANALYSIS OF NUCLEUS-NUCLEUS COLLISION AT HIGH ENERGIES

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Abstract

We propose a novel method for analysis of experimental data obtained at relativistic nucleus-nucleus collisions. The method, based on the ideas of Random Matrix Theory, is applied to detect systematic errors that occur at measurements of momentum distributions of emitted particles. The unfolded momentum distribution is well described by the Gaussian orthogonal ensemble of random matrices, when the uncertainty in the momentum distribution is maximal. The method is free from unwanted background contributions.

Relativistic heavy ion collisions are among major experimental tools that allow to get insight into nuclear dynamics at high excitation energies and large baryon densities. It is expected that in central collisions, at energies that are and will be soon available at SPS(CERN), RHIC(BNL) and LHC(CERN), the nuclear density may exceed by tens times the density of stable nuclei. At such extreme conditions one would expect that a final product of heavy ion collisions could present a composite system that consists of free nucleons, quarks and quark-gluon plasma. However, identification of the quark-gluon plasma, for example, is darken due to a multiplicity of secondary particles created at these collisions. There is no a clear evidence of the quark constituent as well. In fact, there are numerous additional mechanisms of a particle creation that mask the presence of the quark-gluon plasma (QGP). It appears that the QGP could be manifested via the observation of indirect phenomena.

The natural question arises: how to identify a useful signal that would be unambiguously associated with a certain physical process ?

The most popular methods of analysing data produced at relativistic heavy ion collisions are the correlation analysis [1], the analysis of missing masses [2] and effective mass spectra [3], the interference method of identical particles [4]. We recall that results obtained within those methods are sensitive to assumptions made upon the background of measurements and mechanisms included into a corresponding model consideration. As was mentioned above, the larger is the excitation energy, the larger is a number of various mechanisms of the creation that should be taken into account.

As an alternative approach, one could develop a method that should be independent on the background contribution. For instance, there are attempts to use the maximum entropy principle [5], Fourier transform [6] and even by even analysis [7]. Thus, a formulation of a criteria for a selection of meaningful signals is indeed a topical objective of the relativistic heavy ion collisions physics. The major aim of this paper is to suggest a method that does not depend on the background information and relies only upon the fundamental symmetries of the composite system.

Our approach is based on Random Matrix Theory [8] that was originally introduced to explain the statistical fluctuations of neutron resonances in compound nuclei [9] (see also Ref.10). The theory assumes that the Hamiltonian belongs to an ensemble of random matrices that are consistent with the fundamental symmetries of the system. In particular, since the nuclear interaction preserves time-reversal symmetries, the relevant ensemble is the Gaussian Orthogonal Ensemble (GOE). When the time-reversal symmetry is broken one can apply the Gaussian Unitary Ensemble (GUE). The GOE and GUE correspond to ensembles of real symmetric matrices and of Hermitian matrices, respectively. Besides these general symmetry considerations, there is no need in other properties of the system under consideration.

Bohigas *et al* [11] conjectured that RMT describes the statistical fluctuations of a quantum systems whose classical dynamics is chaotic. Quantum spectra of such systems manifest a strong repulsion (anticrossing) between quantum levels, while in non-chaotic (regular) systems crossings are a dominant feature of spectra (see, e.g., [12]). In turn, the crossings are observed when there is no mixing between states that are characterized by different good quantum numbers, while the anticrossings signal about a strong mixing due to a perturba-

tion brought about by either external or internal sources. Nowadays, RMT has become a standard tool for analysing the fluctuations in nuclei, quantum dots and many other systems (see for a review, for example, Ref.13). The success of RMT is determined by the study the statistical laws governing fluctuations having very different origins. Regarding the relativistic heavy ion collision data the study of fluctuation properties of the momentum distribution of emitted particles could provide an information about i)possible errors in measurements and ii)kinematical and dynamical correlations of the composite system.

Let us consider the discrete spectrum $\{E_i\}, i = 1, ..., N$ of a d-dimensional quantum system (d is a number of degrees of freedom). A separation of fluctuations of a quantum spectrum can be based on the analysis of the density of states below some threshold E

$$S(E) = \sum_{i=1}^{N} \delta(E - E_i)\,. \tag{1}$$

We can define a staircase function

$$N(E) = \int_{-\infty}^{E} S(E')dE' = \sum_{i=1}^{N} \theta(E - E'), \tag{2}$$

giving the number of points on the energy axis which are below or equal to E. Here

$$\theta(x) \;=\; \begin{cases} 0 & for\;\; x < 0 \\ 1 & for\;\; x > 1 \end{cases} \tag{3}$$

We separate $N(E)$ in a smooth part $\zeta(E)$ and the reminder that will define the fluctuating part $N_{\text{fl}}(E)$

$$N(E) = \zeta(E) + N_{\text{fl}}(E) \tag{4}$$

The smooth part $\zeta(E)$ can be determined either from semiclassical arguments or using a polynomial or spline interpolation for the staircase function.

To study fluctuations we have to get rid of the smooth part. The usual procedure is to "unfold" the original spectrum $\{E_i\}$ through the mapping $E \rightarrow x$

$$x_i = \zeta(E_i), \qquad i = 1, ..., N \tag{5}$$

Now we can define spacings $s_i = x_{i+1} - x_i$ between two adjacent points and collect them in a histogram. The effect of mapping is that the sequence $\{x_i\}$ has on the average a constant mean spacing (or a constant density), irrespective of the particular form of the function $\zeta(E)$ [14]. To characterize fluctuations one deals with different correlation functions [8]. In this

paper we will use only a correlation function related to spacing distribution between adjacent levels. Below, we follow a simple heuristic argument due to Wigner [15] that illustrates the presence or absence of level repulsion in an energy spectrum.

For a random sequence, the probability that the level will be in the small interval $[x_0 + s, x_0 + s + ds]$ is independent of whether or not there is a level at x_0. Given a level at x_0, let the probability that the next level be in $[x_0 + s, x_0 + s + ds]$ be $p(s)ds$. Then for $p(s)$, the nearest-neighbor spacing distribution, we have

$$p(s)ds = p(1 \in ds|0 \in s)p(0 \in s) \tag{6}$$

Here, $p(n \in s)$ is a probability that the interval of length s contains n levels and $p(n \in ds|m \in s)$ is the conditional probability that the interval of length ds contains n levels, when that of length s contains m levels. One has $p(0 \in s) = \int_s^\infty p(s')ds'$, the probability that the spacing is larger than s. The term $p(1 \in ds|0 \in s) = \mu(s)ds$ [$\mu(s)$ is the density of spacings s], depends explicitly on the choices, 1 and 0, of the discrete variables n, m. As a result, one obtains $p(s) = \mu(s)\int_s^\infty p(s')ds'$ which can be solved to give

$$p(s) = \mu(s)\exp(-\int_0^s \mu(s')ds') \tag{7}$$

The function $p(s)$ and its first moment are normalized to unity,

$$\int_0^s p(s)ds = 1, \qquad \int_0^s sp(s)ds = 1. \tag{8}$$

For a linear repulsion $\mu(s) = \pi s/2$ one obtains the Wigner surmise,

$$p(s) = \frac{\pi}{2}s\exp(-\frac{\pi}{4}s^2), \qquad s \geq 0 \tag{9}$$

For a constant value $\mu(s) = 1$ one obtains the Poisson distribution

$$p(s) = \exp^{-s}, \qquad s \geq 0 \tag{10}$$

As discussed above, when quantum numbers of levels are well defined, one should expect for the spacings the Poisson type distribution, while a Wigner type distribution occurs due to either internal or external perturbations that destroy these quantum numbers. In fact, one of the sources of external perturbations can be attributed to the uncertainty in the determination of the momentum distribution of emitted particles in relativistic heavy ion collisions. We make a conjecture that the above discussed ideas of the RMT are applicable

to the momentum distribution as well. We assume that the momentum distribution may be associated with eigenstates (quantum levels) of a composite system. The difference between energy and momentum is inessential for pions (see below), while we assume that the proton mass should not affect significantly the correlation function.

Another possibilities are the association of the momentum distribution to the spectrum of scattering matrix, or density matrix, which can equally be the object of statistical analysis. Note also, that here we are dealing with the momentum distribution in the target rest frame only, postponing its comparison to that in the center of mass frame, which is more natural for description of interaction. Therefore, we simply replace in Eqs.(1)-(5) the variable E by the variable $|p|$ and construct the corresponding correlation function $p(s)$.

To test the utility and the validity of the proposal we use the experimental data that have been obtained from the 2-m propane bubble chamber of LHE, JINR [16, 17]. The chamber, placed in a magnetic field of 1.5 T, was exposed to beams of light relativistic nuclei at the Dubna Synchrophasotron. Practically all secondaries, emitted at a 4π total solid angle, were detected in the chamber. All negative particles, except those identified as electrons, were considered as π^--mesons. The contaminations by misidentified electrons and negative strange particles do not exceed 5% and 1%, respectively. The average minimum momentum for pion registration is about 70 MeV/c. The protons were selected by a statistical method applied to all positive particles with a momentum of $|p| > 500$ MeV/c (we identified slow protons with $|p| \leq 700$ MeV/c by ionization in the chamber). In this experiment, we had got 20407 ^{12}CC interactions at a momentum of 4.2A GeV/c (for methodical details see [17]) contents 4226 events with more than ten tracks of charged particles. Thus, it was known in advance the accuracy of measurements for available range of the momentum distribution of secondary particles. Consequently, our analysis has been done for different range of values of the momentum distribution to illuminate the degree of the accuracy.

On Fig. 1 the dependence $dN/d|p|$ as a function of the measured momentum (0.15-7.5 GeV/c) of the secondary particles is displayed . The numerical data $N(p)$ were approximated by the polynomial function of the sixth order and we obtain the distribution of various spacings s_i in 2636 events satisfying the condition of χ^2 per degree of freedom less than 1.0. Momenta are well defined in the region 0.15-1.14 GeV/c (region I, Fig. 2a), where the minimal value of the proton momentum is 0.15 GeV/c. The intermediate region (region II, Fig. 2b) covers the values 1.14-4.0 GeV/c. The region 4.0-7.5 GeV/c is the less reliable

one (Fig. 2c). The spacing probability nicely reproduces this tendency depending on the region of the momentum distribution. The function $p(s)$ has the Poisson distribution for the region I, where the momentum distribution was defined with a high accuracy. The region II corresponds to the intermediate situation, when the spacing distribution lies between the Poisson and the Wigner distributions. The less reliable region of the values has a Wigner type distribution for the spacing probability (Fig. 2c). Indeed, the distribution reflects a strong deviation from the regular behavior, observed for the measurements with a high degree of the accuracy.

Summarizing, we propose a method to analyse data obtained at relativistic heavy ion collisions. The method does not depend on the background of the measurements and provides a reliable information about correlations brought about by external or internal perturbations. In particular, we demonstrate that the method manifests the perturbations due to the uncertainty in the determination of the momentum distribution of secondary emitted particles.

Acknowledgments

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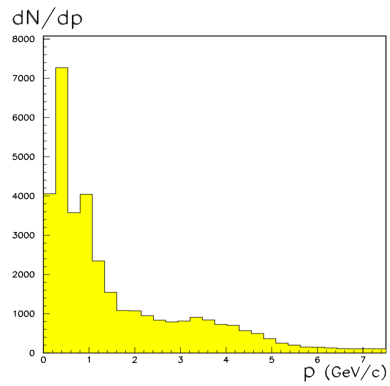

Fig.1 $dN/d|p|$ as a function of the measured momentum of the secondary particles.

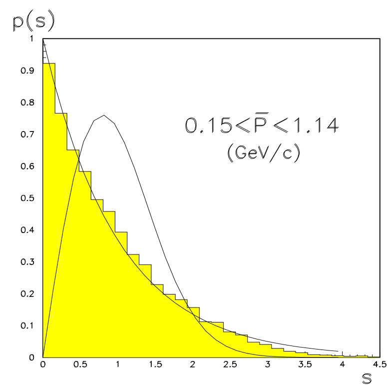

Fig.2 a

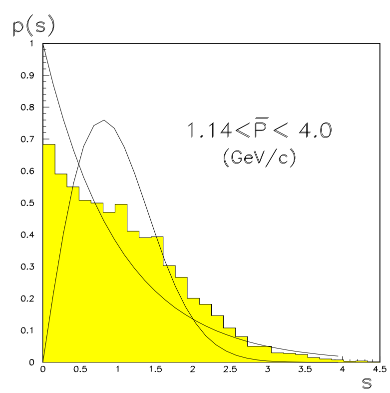

Fig.2 b

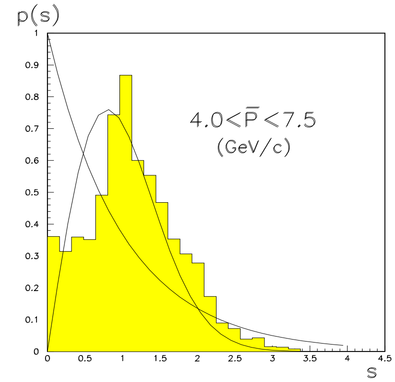

Fig.2 c Nearest-neighbor spacing momentum distribution $p(s)$ for different regions of measured momenta: a)$0.15 < |p| < 1.14$ GeV/c; b)$1.14 < |p| < 4.0$ GeV/c; c)$4.0 < |p| < 7.5$ GeV/c. The solid line is the Wigner-Dyson distribution and the dashed line is the Poisson distribution.